# Hypothesis Testing with Two Samples

## *Independent Samples*

## *Commonly Used*

Compare methods of teaching reading

(whole language vs. phonics)

Compare drug treatment effectiveness

(drug effect vs. sugar pill)

Compare methods of treatment

(antidepressant drug vs. counseling)

Compare differences between groups

(test anxiety – male vs. female)

*Example:* You want to determine whether or not there's a difference between the drinking habits of fraternity members and non-fraternity members.

## Assumptions of the Independent Samples t-test

- Variable has normal distribution
- Variances of the groups are equivalent (homogeneous)
- Samples are randomly sampled

Randomly select members of a fraternity and non-fraternity. Record the number of beers consumed in a week.

| Frat | NonFrat |
|---|---|
| 6 | 0 |
| 3 | 5 |
| 2 | 3 |
| 4 | 4 |
| 5 | 3 |
| 6 | 6 |
| 7 | 3 |
| 5 | 6 |
| 4 | 5 |
| 5 | 4 |
| 4 | 4 |
| 8 | 2 |
| 6 | |
| 7 | |

Study based on a sample of 14 frat members and 12 non-frat members ($n_1$=14, $n_2$=12)

$\text{Mean}_1$ = 5.14 $s_1$ = 1.66

$\text{Mean}_2$ = 3.75 $s_2$ = 1.71

There is a difference in the means, but is the difference significant?

Formulate the Null Hypothesis:

*There is no significant difference in the drinking habits of fraternity members compared to non-fraternity members.*

Calculate the Estimate of the Standard Error

$$s_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}\bullet\left[\frac{1}{n_1}+\frac{1}{n_2}\right]}$$

$$s_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{(14-1)2.76+(12-1)2.92}{14+12-2}\bullet\left[\frac{1}{14}+\frac{1}{12}\right]}$$

$$s_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{(13)2.76+(11)2.92}{24}\bullet[0.07+0.08]}$$

$$s_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{35.88+32.12}{24}\bullet[0.15]}$$

$$s_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{68}{24}\bullet[0.15]}$$

Calculate the Estimate of the Standard Error

$$s_{\bar{x}_1-\bar{x}_2} = \sqrt{2.83[0.15]}$$

$$s_{\bar{x}_1-\bar{x}_2} = \sqrt{0.42}$$

$$s_{\bar{x}_1-\bar{x}_2} = 0.65$$

Std Error = 0.65

The ***t ratio*** formula

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{s_{\bar{x}_1-\bar{x}_2}}$$

$X_1 = 5.14$

$X_2 = 3.75$

The ***t ratio*** formula

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{s_{\bar{x}_1 - \bar{x}_2}}$$

$$t = \frac{(5.14 - 3.75)}{0.65}$$

$$t = \frac{1.39}{0.65}$$

$$t = 2.14$$

Determine Critical Value

$df = n_1 + n_2 - 2$

$= 14 + 12 - 2$

$= 24$

($\alpha = .05$)

*Family of t Distributions (Two-Tailed Test)*

| Degrees of Freedom (df) | LEVEL OF S .20 | .10 | .05 |
|---|---|---|---|
| 5 | 1.476 | 2.015 | 2.571 |
| 6 | 1.440 | 1.943 | 2.447 |
| 7 | 1.415 | 1.895 | 2.365 |
| 8 | 1.397 | 1.860 | 2.306 |
| 9 | 1.383 | 1.833 | 2.262 |
| 10 | 1.372 | 1.812 | 2.228 |
| 11 | 1.363 | 1.796 | 2.201 |
| 12 | 1.356 | 1.782 | 2.179 |
| 13 | 1.350 | 1.771 | 2.160 |
| 14 | 1.345 | 1.761 | 2.145 |
| 15 | 1.341 | 1.753 | 2.131 |
| 16 | 1.337 | 1.746 | 2.120 |
| 17 | 1.333 | 1.740 | 2.110 |
| 18 | 1.330 | 1.734 | 2.101 |
| 19 | 1.328 | 1.729 | 2.093 |
| 20 | 1.325 | 1.725 | 2.086 |
| 21 | 1.323 | 1.721 | 2.080 |
| 22 | 1.321 | 1.717 | 2.074 |
| 23 | 1.319 | 1.714 | 2.069 |
| 24 | 1.318 | 1.711 | 2.064 |
| 25 | 1.316 | 1.708 | 2.060 |
| 26 | 1.315 | 1.706 | 2.056 |
| 27 | 1.314 | 1.703 | 2.052 |
| 28 | 1.313 | 1.701 | 2.048 |
| 29 | 1.311 | 1.699 | 2.045 |
| 30 | 1.310 | 1.697 | 2.042 |
| 40 | 1.303 | 1.684 | 2.021 |
| 50 | 1.299 | 1.676 | 2.009 |
| 60 | 1.296 | 1.671 | 2.000 |
| 80 | 1.292 | 1.664 | 1.990 |
| 100 | 1.290 | 1.660 | 1.984 |
| 120 | 1.289 | 1.658 | 1.980 |
| ∞ | 1.282 | 1.645 | 1.960 |


| Freedom (df) | .20 | .10 | .05 |
|---|---|---|---|
| 5 | 1.476 | 2.015 | 2.571 |
| 6 | 1.440 | 1.943 | 2.447 |
| 7 | 1.415 | 1.895 | 2.365 |
| 8 | 1.397 | 1.860 | 2.306 |
| 9 | 1.383 | 1.833 | 2.262 |
| 10 | 1.372 | 1.812 | 2.228 |
| 11 | 1.363 | 1.796 | 2.201 |
| 12 | | | 2.179 |
| 13 | | | 2.160 |
| 14 | | | 2.145 |
| 15 | | | 2.131 |
| 16 | | | 2.120 |
| 17 | 1.333 | 1.74 | 2.110 |
| 18 | 1.330 | 1.73 | 2.101 |
| 19 | 1.328 | 1.729 | 2.093 |
| 20 | 1.325 | 1.725 | 2.086 |
| 21 | 1.323 | 1.721 | 2.080 |
| 22 | 1.321 | 1.717 | 2.074 |
| 23 | 1.319 | 1.714 | 2.069 |
| 24 | 1.318 | 1.711 | 2.064 |
| 60 | 2.390 | 2.660 | 3.460 |
| 80 | 2.374 | 2.639 | 3.416 |
| 100 | 2.364 | 2.626 | 3.390 |
| 120 | 2.358 | 2.617 | 3.373 |
| ∞ | 2.327 | 2.576 | 3.291 |


critical value = 2.064

The *t ratio* formula

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{s_{\bar{x}_1 - \bar{x}_2}}$$

$$t = \frac{(5.14 - 3.75)}{0.65}$$

$$t = \frac{1.39}{0.65}$$

$$t = 2.14$$

Determine Critical Value

$df = n_1 + n_2 - 2$

$= 14 + 12 - 2$

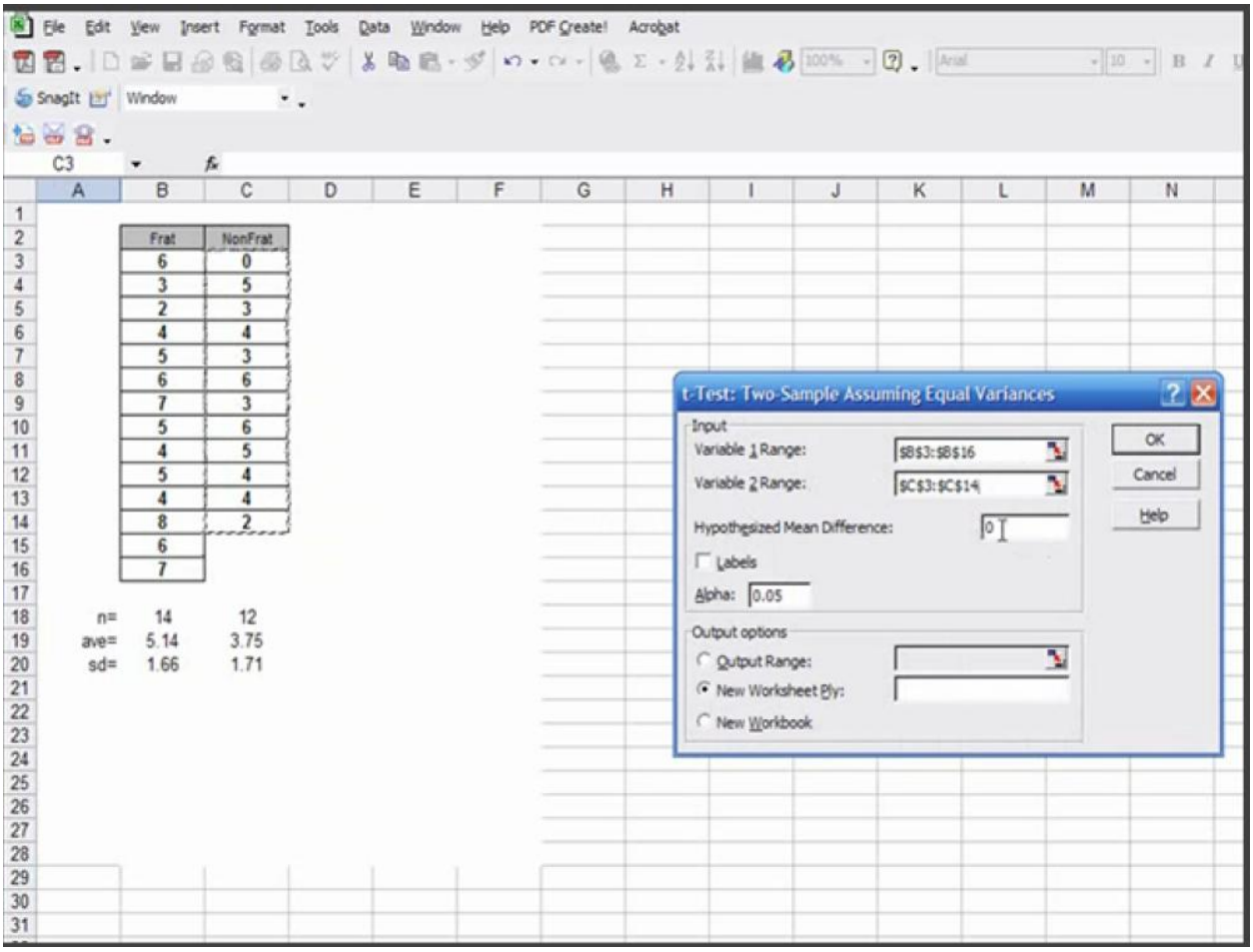
$= 24$

$(\alpha = .05)$

Critical value = **2.06**

*t ratio* = **2.14**

Hence, reject the null.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C3
Frat NonFrat
6 0
3 5
2 3
4 4
5 3
6 6
7 3
5 6
4 5
5 4
4 4
8 2
6
7
n= 14 12
ave= 5.14 3.75
sd= 1.66 1.71
t-Test: Two-Sample Assuming Equal Variances
Input
Variable 1 Range: $B$3:$B$16
Variable 2 Range: $C$3:$C$14
Hypothesized Mean Difference: 0
Labels
Alpha: 0.05
Output options
Output Range:
New Worksheet Ply:
New Workbook
OK
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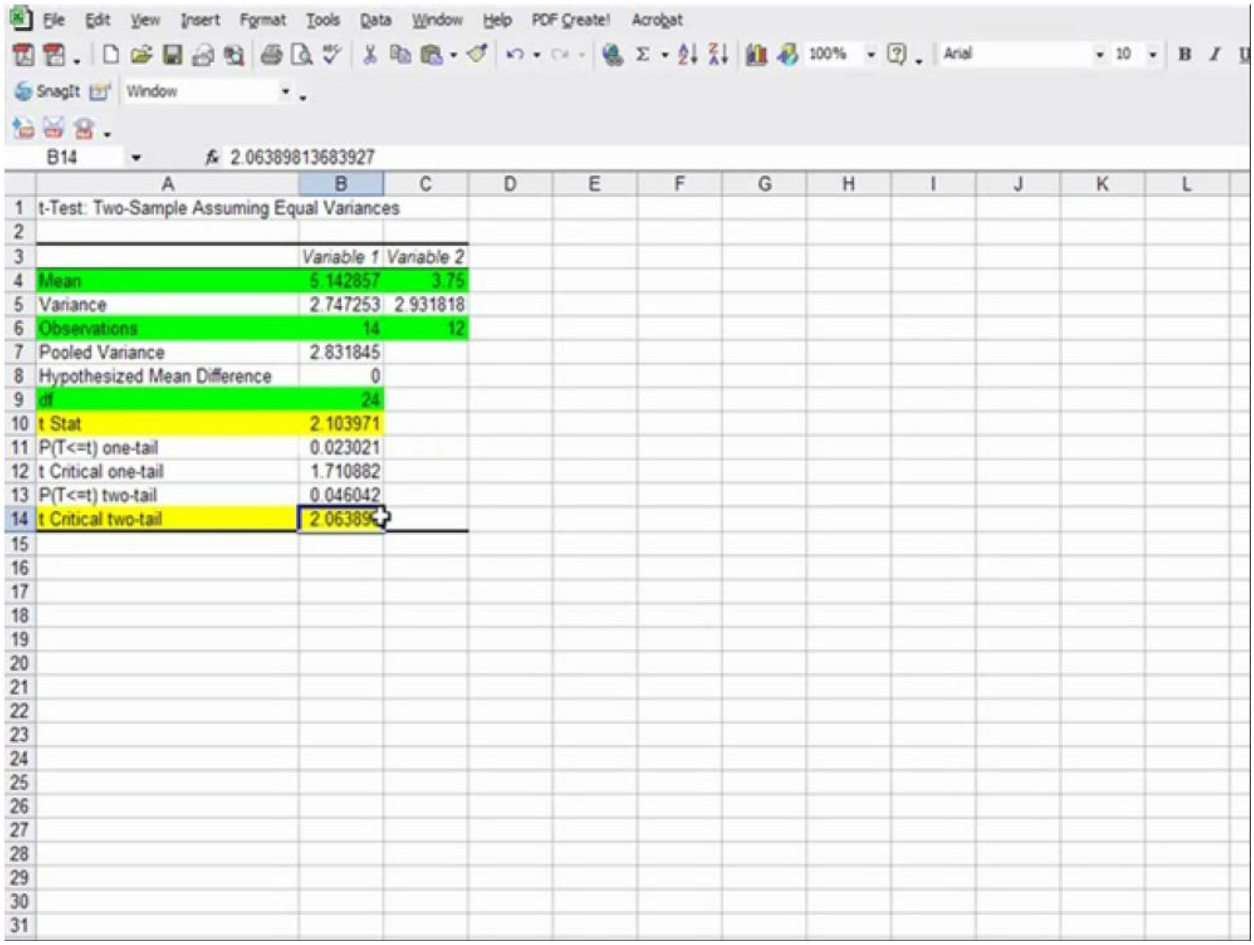
Help




| | A | B | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | t-Test: Two-Sample Assuming Equal Variances | | | | | | | | | | | |
| 2 | | | | | | | | | | | | |
| 3 | | *Variable 1* | *Variable 2* | | | | | | | | | |
| 4 | Mean | 5.142857 | 3.75 | | | | | | | | | |
| 5 | Variance | 2.747253 | 2.931818 | | | | | | | | | |
| 6 | Observations | 14 | 12 | | | | | | | | | |
| 7 | Pooled Variance | 2.831845 | | | | | | | | | | |
| 8 | Hypothesized Mean Difference | 0 | | | | | | | | | | |
| 9 | df | 24 | | | | | | | | | | |
| 10 | t Stat | 2.103971 | | | | | | | | | | |
| 11 | P(T<=t) one-tail | 0.023021 | | | | | | | | | | |
| 12 | t Critical one-tail | 1.710882 | | | | | | | | | | |
| 13 | P(T<=t) two-tail | 0.046042 | | | | | | | | | | |
| 14 | t Critical two-tail | 2.06389 | | | | | | | | | | |
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